ET'nA 2017 - Encounter in Topology 'n Algebra

This is the list of the abstracts of the talks.

Bruno Benedetti and Matteo Varbaro,
(Summer school talk) Polytopes, Dual Graphs, and Line Arrangements
Polytopes (like cubes, pyramids, prisms...) have always been studied since the beginning of mankind. They exist in nature and are replicated by architects. In the past century, with the advent of computers, they've regained central interests also in connection with optimization problems and digital data. We will focus on the notion of the "graph" of the polytope, namely, the structure formed by its vertices and edges. This apparently easy subject hides important open questions, like the polynomial Hirsch conjecture. It also hides a fascinating connection with the intersection graphs of arrangements of lines. This opens bridges to old and new techniques in commutative algebra and algebraic geometry.

Christin Bibby, Representation stability and arrangements
For certain sequences of groups (eg. symmetric groups, Weyl groups, or some other types of complex reflection groups), we are interested in an associated sequence of arrangements of subvarieties. The complement to such an arrangement is akin to an ordered configuration space, and its rational cohomology can be viewed as a representation of the corresponding group. Here we see the phenomenon of representation stability, which in particular means that upon decomposing the representations in our sequence into irreducibles, the multiplicity of each irreducible representation is eventually constant. A consequence of representation stability is that the orbit space (eg. the unordered configuration space) is (rationally) homologically stable. We study the stability of our sequence of representations as a consequence of combinatorial stability of the arrangement.

Ted Chinburg, Negative curves on surfaces and hyperbolic codes
In this talk I will begin by describing some of what is known about spherical codes. These are sets of points on a unit Euclidean n-sphere which are at least a prescribed minimal distance apart. This subject includes, for example, the study of kissing numbers of spheres. I'll then describe a counterpart in hyperbolic space, and an application of such "hyperbolic codes" to studying curves of small genus and negative self intersection on surfaces. This is joint work with Matt Stover.

Suyoung Choi, Finite group action on real toric variety and its geometric representations

The fundamental theorem of toric geometry says that the class of toric varieties bijectively correspond to the class of fans. This correspondence has opened up fertile research areas which reveal rich interactions between geometry and combinatorics. For example, many studies on the representations of Weyl groups on the cohomology oftoric varieties associated to root systems are found in the literature such as Procesi(1990),Dolgachev-Lunts(1994), and Stembridge(1994). The real locus $X^{\mathbb{R}}$ *of a toric variety* X*, which is the fixed point set of the canonical involution, forms a real subvariety, and it is called a real toric variety. While many arguments for complex toric varieties apply verbatim, it turns out that real toric varieties sometimes exhibit essentially different nature. In this talk, we develop a framework to construct geometric representations of finite groups* G *through the correspondence between real toric variety* $X^{\mathbb{R}}$ *and its mod* 2*-fan. In particular, we give a combinatorial description of the* G*-module structure of the homology of* $X^{\mathbb{R}}$*. As applications, we make explicit computations of the Weyl group representations on the homology of real toric varieties associated to the Weyl chambers of classical types, which show an interesting connection to the topology of posets. This talk is based on a joint work with Soojin Cho and Shizuo Kaji. (https://arxiv.org/abs/1704.08591)*

Daniel C. Cohen, Topological complexity of surfaces

Topological complexity is a numerical homotopy-type invariant introduced by Farber about 15 years ago, motivated by the motion planning problem from robotics. For a space X, this invariant TC(X), the sectional category of the fibration sending a path in X to its two endpoints, provides a measure of the complexity of navigation in X. Computing TC(X) is sometimes easy, sometimes hard. I will attempt to illustrate this, primarily with surfaces.

Alexandru Constantinescu, Linear syzygies, hyperbolic Coxeter groups, and regularity

This talk is based on a surprising connection between a question in commutative algebra regarding Castelnuovo-Mumford regularity, and a question of Gromov about hyperbolic Coxeter groups. We show that the virtual cohomological dimension of a Coxeter group is essentially the same as the Castelnuovo-Mumford regularity of the Stanley-Reisner ring of its nerve. Using this relation, we modify a construction of Osajda in group theory to find for every positive integer r *a monomial ideal generated in degree two, with linear syzygies, and regularity of the quotient equal to* r*. Previously known examples had regularity less than 5. For Gorenstein ideals we prove that the regularity of their quotients can not exceed four, thus showing that for* $d > 4$ *every triangulation of a d-manifold has a hollow square or simplex. We also show that for most monomial ideals generated in degree two and with linear syzygies the regularity is* $O(log(log(n))$*, where n is the number of variables, improving in this case a bound found by Dao, Huneke and Schweig. All results are in collaboration with with Thomas Kahle and Matteo Varbaro.*

Emanuela De Negri, Nice simplicial complexes associated to classical determinantal ideals

Let I be a homogeneous ideal in a polynomial ring R over a field K. Since passing to an initial ideal is a at deformation, one can deduce many information on the ideal from the initial ideal. If the initial ideal is square free, one associates to it a simplicial complex and get information by studying the combinatorics of the complex. In this talk we introduce this methods, and then we give two different applications to classical determinantal ideals. We first consider cogenerated ideals of Pfaffians in a skew-symmetric matrix of indeterminates. We characterize cogenerated Pfaffian ideals whose natural generators are a Gröbner basis with respect to any anti-diagonal term order, and we study their associated simplicial complexes. We also get a formula for computing the multiplicity of such Pfaffian ideals. Then we consider the ideals generated by the minors of a fixed size t in an $n \times n$ matrix of indeterminates. Such ideals define a Gorenstein ring if and only if $n - t$ is even. It is well known that the generators form a Gröbner basis w.r.t. the diagonal order. The corresponding initial ideal is Cohen-Macaulay, but not Gorenstein. In the case $n - t = 2$, we consider a different, apparently less natural, term order and we get a Gorenstein initial ideal. Key point of this result is the fact the associated simplicial complex is isomorphic to the matching complex of a special graph.

Graham Denham and Alex Suciu,
(Summer school talk) Resonance varieties

In its simplest form, the Bernstein-Gelfand-Gelfand (BGG) correspondence associates a chain complex of free graded modules over a polynomial algebra to a module over an exterior algebra in way that generalizes the classical Koszul complex. The resonance varieties measure deviations from exactness of this chain complex in a concrete, geometric fashion. In these introductory lectures, we will explain the algebraic notions underlying these constructions, and present some structural results. We will illustrate the general theory with a number of examples, such as the computation of resonance varieties of the Stanley-Reisner ring associated to a simplicial complex, or of the Orlik-Solomon algebra associated to a hyperplane arrangement. We will indicate how computer algebra can help investigate resonance varieties explicitly, using Macaulay2, and we will conclude with a brief overview of some topics of current research.

Luca Moci, Quasi-polynomial invariants of CW-complexes, product of arithmetic matroids, and convolution formula

In a recent series of papers by various authors, the theory of colorings and flows on graphs has been extended to the higher-dimensional case of CW-complexes. After recalling the main definitions, we show that the dichromate function of CW-complexes is the so-called Tutte quasi-polynomial, and that the "modified Tutte-Krushkal-Renhardy polynomials" are actually arithmetic Tutte polyniomials. In order to prove that, we define a new operation, the product of arithmetic matroids (joint work with Emanuele Delucchi). Then a natural question is how the arithmetic Tutte polynomial of two arithemetic matroids relates to that of their product. The answer is given by a convolution formula, which generalizes several known results and is related to a Hopf algebra structure.

Alfio Ragusa, Weak Lefschetz and Betti Lefschetz algebras

This is a survey on some works by G. Zappalá and my-self in which the Weak Lefschetz Property (WLP) for Artinian standard graded algebras is investigated. In particular, it is shown that the Hilbert function of an almost complete intersection Artinian standard graded algebra of codimension 3 is a Weak Lefschetz sequence, i.e. it is the Hilbert function of some Artinian algebra with WLP or equivalently it is unimodal and the positive part of their first differences is a O-sequence. Moreover we give both some numerical condition on the Hilbert function and other conditions on the graded Betti numbers in order to force Artinian Gorenstein standard graded algebras of codimension 3 to enjoy the WLP. For Artinian standard graded algebras with the WLP we study the behavior of their linear quotients both with respect to the Hilbert function and to the graded Betti numbers. From this we produce a new property denominated Betti Weak Lefschetz Property (-WLP) which permits a good behavior of the grade Betti numbers for the linear quotients of Artinian standard graded algebras with the WLP. We find conditions on the generators' degrees of a complete intersection Artinian graded algebra with the WLP which force the algebra to have the -WLP.

Yury Ustinovskiy, On face numbers of flag simplicial complexes

Denham, Suciu and Panov, Ray computed ranks of homotopy groups and Poincare series of a moment-angle-complex and a Davis-Januzskiewicz space associated to a flag simplicial complex K. In this talk we revisit these results and interpret them as polynomial bounds on the face numbers of an arbitrary simplicial flag complex.

Masahiko Yoshinaga, G–Tutte polynomials

Given a list of integer vectors, one can associate several mathematical objects, e.g., matroids, hyperplane arrangements, toric arrangements, zonotopes etc. Each has certain (quasi)polynomial invariants which possesses rich topological and enumerative information. Among others the Tutte polynomial detects Betti numbers of the complement of a complex hyperplane arrangement and the arithmetic Tutte polynomial acts similarly for toric arrangements. In this talk, we introduce G-Tutte polynomial for an abelian group G (with a weak assumption on the finiteness of torsions). Main examples are abelian Lie groups with finitely many connected components. It is a generalization of "Tutte polynomials" in the sense that $G = C$ *and* C^* *recovers Tutte and arithmetic Tutte polynomial, respectively. We see that many well known properties are shared also by G-Tutte polynomials. We also discuss the topology of the complement of corresponding "arrangements" for non-compact group G. This is a joint work with Ye Liu and Tan Nhat Tran (Hokkaido).*